NASA Contractor Report 198054

Yeh-Stratton Criterion for Stress Concentrations on Fiber-Reinforced Composite Materials

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NASA Contract No. NCC 4-107
November 1996

National Aeronautics and
Space Administration

NASA Contractor Report 198054

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Abstract

This study investigated the Yeh-Stratton Failure Criterion with the stress concentrations on fiber-reinforced composites materials under tensile stresses. The Yeh-Stratton Failure Criterion was developed from the initial yielding of materials based on macromechanics. To investigate this criterion, the influence of the materials anisotropic properties and far field loading on the composite materials with central hole and normal crack were studied. Special emphasis was placed on defining the crack tip stress fields and their applications. The study of Yeh-Stratton criterion for damage zone stress fields on fiber-reinforced composites under tensile loading was compared with several fracture criteria; Tsai-Wu Theory, Hoffman Theory, Fischer Theory, and Cowin Theory. Theoretical predictions from these criteria are examined using experimental results.

Introduction

The Yeh-Stratton criterion was developed from a yielding criterion that was modified to a generalized failure criterion and applied to fiber-reinforced composites with a central crack or a circular cutout under tensile loadings. The purpose of a material failure criterion is to establish a theoretical margin of safety that has been validated by experiments. Obtaining the crack tip stress field is necessary before a failure criterion of composite materials is applied. However, the crack tip stress field is complicated because numerous factors influence crack tip stress distribution. Understanding factors that contribute to crack tip stress fields is of critical importance in analyzing composite laminates.

Stress Concentration of Composites

First, the stress concentration in fiber-reinforced composites with tensile loadings is discussed. In fracture analysis, it is significant to consider the crack tip stress field. Lekhnitskii extended Muskhelishvili's work in the plane theory of isotropic elasticity to the anisotropic case.[1] Later, Sih et al[2] derived the stress field equations for the anisotropic elastic body with a crack. However, the crack tip stress field equations established by Lekhnitskii and Sih were in a complex form. Theocaris and Philippidis modified these stress field equations into a real number solution.[3] These crack tip stress field equations will be applied to the following discussion and substituted into the failure criteria to analyze fracture situations of composites with cracks.

Fracture Criteria of the Composite Materials

Once the stress distribution of a composite is calculated, the point of interest should be examined with a failure criterion to determine whether the composite will fail or not. In reference 4 the Yeh-Stratton criterion was generalized for fibrous composites. Although the generalized Yeh-Stratton criterion was developed for uncracked bodies, it should apply to cracked bodies of anisotropic materials because the criterion is based on continuum mechanics. Some other relevant criteria to be compared are the Tsai-Wu theory, Hoffman theory, Fischer criterion, and Cowin criterion.[5]

Considerable efforts have been devoted to the formulation of composite failure criteria and to their correlation with experimental data, but no criterion has been fully adequate. The analyses of the Hoffman, Fischer, and Cowin theories show them to be valid only under special cases. This limits their direct application to general materials.

Results and Conclusions

In this study, experimental results obtained by Tan[6] for AS4/3502 $[\pm\theta_2]_s$ family of laminates with central holes and normal cracks were examined using the Yeh-Stratton criterion and several other failure theories. The fracture of AS4/3502 $[\pm\theta_2]_s$ is a mixed-mode type and dominated by the epoxy matrix. The experimental results and geometric dimensions of the samples for the graphite/epoxy AS4/3502 $[\pm\theta_2]_s$ from Tan[6] are borrowed. Each group of samples has five circular hole diameters in various plate widths as well as five normal crack lengths in various plate widths. Figures 1 and 2 display the predictions from the Yeh-Stratton criterion, the Tsai-Wu theory, and experimental data. The prediction curves from both theories are very close. These curves indicate that the Yeh-Stratton criterion is reliable and accurate as compared with the most popular strength criterion. Table 1 shows an error analysis of this study. For the Yeh-Stratton criterion, the predictions of failure stresses are off by 10 percent. These criteria produce good results with this AS4/3500 $[\pm\theta_2]_s$ material. However, the Yeh-Stratton criterion offers more accurate predictions than the various other theories (fig. 1).

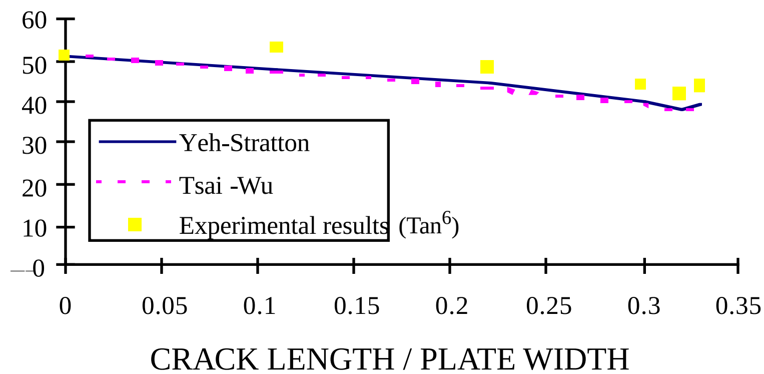

Figure 1. Comparison of the Yeh-Stratton, the Tsai-Wu theory, and experimental data for AS4/3502 $[\pm 30°]_s$ with a center crack.

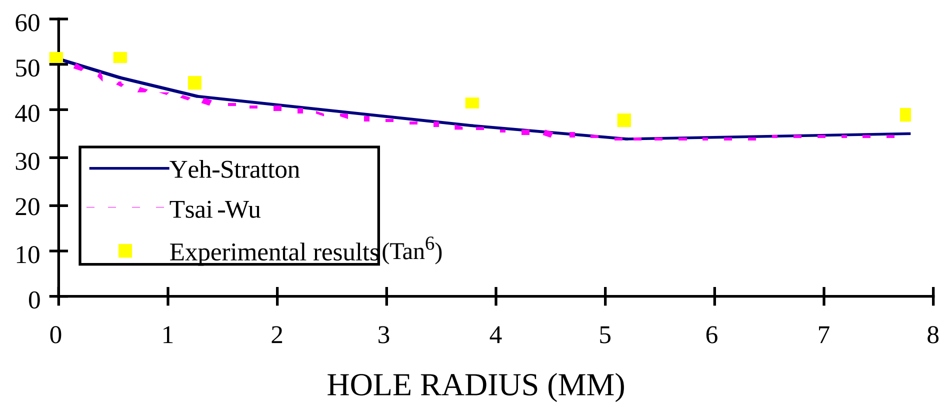

Figure 2. Comparison of the Yeh-Stratton, the Tsai-Wu theory, and experimental data for AS4/3502$[\pm 30°]_s$ with a circular hole.

Table 1. Error analysis of the data of AS4/3502 $[\pm\theta_2]_s$ from different theories.

Theory	Center crack			Circular hole		
	30°	45°	60°	30°	45°	60°
Yeh-Stratton	10.0%	10.0%	10.2%	10.0%	9.9%	10.0%
Tsai-Wu	11.0%	11.0%	10.7%	10.0%	10.0%	10.2%
Cowin	10.5%	10.6%	10.6%	10.3%	10.2%	11.0%
Hoffman	10.4%	10.6%	10.6%	10.3%	10.2%	11.0%
Fischer	11.0%	11.0%	10.8%	12.0%	11.0%	11.0%

For a constant crack length or a constant circular hole diameter on the AS4/3502 $[\pm\theta_2]_s$ composite plate, the required externally applied failure stress increases when the distance from the crack tip to the initial failure point increases. The influence from the existing cracks or holes to the material failure is, therefore, a localized effect. However, for a constant distance from the crack tip to the initial failure point, the applied failure stress is in inverse proportion to the crack lengths or hole diameters. In other words, the failure stress reduces when the crack lengths or the hole diameters increase. This reduction indicates that the existing cracks or holes will reduce the strength of materials.

One basic assumption in the theory of linear elastic fracture mechanics (LEFM) is that the distance from the crack tip to the initial failure point i.e. size of the plastic zone or the damage zone is much less than the crack length. However, the distances from the crack tip to the initial failure point of the composite material AS4/3502 $[\pm 60]_s$ are about the same order of magnitude as their crack lengths, for example 9.6 mm to 15.14 mm, 5.76 mm to 10.14 mm and 4.16 mm to 7.62 mm etc. (table 2). The violation of the basic assumption in LEFM theory presents a challenging task in the fracture research of composite materials. The distance from a crack tip or a cutout to the initial failure point is increased as the direction of the fiber angle increased. This behavior is caused by the increase of the fiber direction angle. The material needs a larger damage zone in order to cover the larger region with lower stress, making the distance from crack tip to the initial failure point increased to about the same order of magnitude as their initial crack lengths.

Table 2. The distance from the crack tip to the initial failure point of AS4/3502 $[\pm\theta_2]_s$ with a central crack.

30° (mm)	Crack length (mm)	45° (mm)	Crack length (mm)	60° (mm)	Crack length (mm)
0.018	1.35	0.096	1.47	0.744	1.29
0.03	2.67	0.136	2.74	1.32	2.59
0.068	7.47	0.324	7.54	4.16	7.62
0.084	10.03	0.4	10.06	5.76	10.14
0.136	15.16	0.64	15.16	9.6	15.14

Future Activities

Better understanding of the failure or damage of composite materials will provide a more reliable design methodology in the applications of advanced composite materials. To further understand this research subject, more experimental data as well as theoretical model development are required.

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REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY (Leave blank)

2. REPORT DATE
November 1996

3. REPORT TYPE AND DATES COVERED
Contractor Report

4. TITLE AND SUBTITLE
Yeh-Stratton Criterion for Stress Concentrations on Fiber-Reinforced Composite Materials

5. FUNDING NUMBERS
WU-505-63-84

6. AUTHOR(S)
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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
NASA Dryden Flight Research Center
P.O. Box 273
Edwards, California 93523-0273

8. PERFORMING ORGANIZATION REPORT NUMBER
H-2141

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
National Aeronautics and Space Administration
Washington, DC 20546-0001

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
NASA CR-198054

11. SUPPLEMENTARY NOTES
Technical monitor: W. Lance Richards. Hsien-Yang Yeh, Professor, Mechanical Engineering, California State University, Long Beach, California 90840.

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Unclassified—Unlimited
Subject Category 24

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

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14. SUBJECT TERMS
Composite materials; Composite research; Failure theory; Macromechanics; Stress concentrations

15. NUMBER OF PAGES
9

16. PRICE CODE
AO2

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	Unlimited

NSN 7540-01-280-5500

Available from the NASA Center for AeroSpace Information, 800 Elkridge Landing Road, Linthicum Heights, MD 21090; (301)621-0390

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. Z39-18
298-102